APPLICATION OF FUZZY ANALYTIC HIERARCHY PROCESS FOR CONTRACTOR SELECTION PROBLEM

Nur Syuhaila Abdul Rashid1*, Nur Amirah Mohd Amin1, Nor Faradilah Mahad1

1Faculty of Computer and Mathematical Sciences (FSKM)
Universiti Teknologi MARA (UiTM) Cawangan Negeri Sembilan, Kampus Seremban,
Persiaran Seremban Tiga/1, Seremban 3, 70300, Seremban, Negeri Sembilan

*Corresponding author: nrsyuhaila97@gmail.com

Abstract
Choosing the best contractor for services in the manufacturing company is one of the most important things in decision making processes. Nevertheless, it is a complex issue since the selection of best contractor is usually a multi criteria decision making problem. Thereby, the decision maker’s knowledge is often vague and inaccurate. The objective of this study is to select the best maintenance contractor for a computer hard disk drive manufacturer and data storage company using Fuzzy Analytic Hierarchy Process (FAHP) method. The process of evaluating and selecting the best contractor will be based on multiple criteria that are often in conflict. Five criteria which are price (C1), past performance (C2), technical skills (C3), availability (C4) and financial stability (C5) of the contractors were used to select the best contractor from a list of alternatives (A1, A2, A3 and A4) provided by the decision makers. Throughout this study, the result shows that C3 is the most preferred criterion and the ranking order is C3 ≻ C2 ≻ C1 ≻ C4 ≻ C5. Simultaneously, the ranking order for alternative is A4 ≻ A3 ≻ A2 ≻ A1 as contractor A4 is selected as the best contractor to handle the machinery maintenance support for the company. Overall, the proposed FAHP method is beneficial as it allows the decision maker(s) to use their judgments are in the form of linguistic variables in selecting the most preferable alternative or option based on the existing multiple criteria.

Keyword: Analytic Hierarchy Process (AHP), Contractor selection, Decision making, Fuzzy AHP

Introduction
A contractor is someone who is appointed to perform any services which are required by their client. In this context, most of the clients which are usually the manufacturing companies, have a large number of machines and they might have to consider hiring a contractor for maintenance (Burhanuddin, Halawani, Ahmad, & Tahir, 2010). However, being the most desirable maintenance contractor usually varies significantly depending on the machine that needs to be maintained. Therefore, in order to be the best contractor, they must have relevant features as well as extensive working experience dealing with machine tools (Burhanuddin et al., 2010).

It is apparent that the contractor selection includes numerous attributes and most of those attributes are qualitative factors that require human experts to assess subjectively (Rahimdel & Ataei, 2014). In this context, the contractor selection problem can be seen as a Multi Criteria Decision Making (MCDM) problem. MCDM is an efficient approach for ranking by a specific number of alternatives with respect to multiple criteria (Rahimdel & Ataei, 2014). One of the popular MCDM methods is Analytic Hierarchy Process (AHP). AHP was developed by Saaty (Rahimdel & Ataei, 2014). It is a theory of measurement derived by pairwise comparison and the priority scales is constructed based on the judgment of experts.
(Rahimdel & Ataei, 2014). By using the scale of absolute judgment that represents the amount of one element influencing another with respect to a given attribute, the comparison is made (Aydin & Kahraman, 2013). However, the complexity and uncertainty of the objective things make the information given by the experts that usually comprises linguistic form of variables and fuzzy judgment rather than crisp evaluations (Wang, Qian, & Feng, 2011). Thus, the concept of AHP method is extended to the Fuzzy Analytic Hierarchy Process (FAHP) where the ambiguity is considered (Wang et al., 2011). FAHP method is the combination of the concepts of fuzzy set theory and hierarchical structure analysis and this method helps to solve MCDM problems in a more systematic way (Wang et al., 2011). FAHP method applies the similar principle as AHP comparison pairwise function, but, instead of using a crisp number judgments, it uses fuzzy number in the comparison of judgments (Aydin & Kahraman, 2013).

In this study, the FAHP method is applied to assess and select the contractors based on the existing multiple criteria that are often in conflict. Thus, the objective of this study is to select the best maintenance contractor for a computer hard disk drive manufacturer and data storage company by using FAHP method. This paper contains six sections which are (1) Introduction, (2) Literature Review, (3) Materials and Methods, (4) Result and Discussion, (5) Conclusion and lastly (6) Acknowledgment.

**Literature Review**

Zadeh (1965) proposed the set theory of fuzzy to consider the uncertainty of human in making decisions. FAHP is used to define a subjective pair wise comparison judgments of both criteria and the alternatives through linguistic variable (Ayca & Hasan, 2017). A strict mathematical framework is used in the theory so that a vague phenomenon can be studied. Fuzzy set existed with a continuity of membership functions. The function assigns a grade of membership to each object that associated with each fuzzy set (Aydin & Kahraman, 2013). The degree of membership is derived with a range of [0, 1] (Ayca & Hasan, 2017). Fuzzy number in FAHP can be represented by triangular or trapezoidal set of numbers. However, fuzzy numbers are often represented by using Triangular Fuzzy Numbers (TFNs). **Figure 1** shows the parameters of \( l, m, u \) representing the lower value, mid-point value and upper value of the fuzzy description system respectively (Ayca & Hasan, 2017).

**Figure 1** Graphical Representation of Triangular Membership Function

Buckley (1985) proposed a new method of FAHP by using the geometric mean. Geometric mean is a uniquely appropriate rule for aggregating the decision maker’s judgements within the AHP method as it can maintain the reciprocal property in the combined pairwise comparison matrix (Liu et al., 2017). The other popular approach used in FAHP is Extent Analysis method by Chang (1996). Chang (1996) utilised triangular fuzzy numbers for the
pairwise comparison scale of FAHP and the scale analysis method for the pairwise synthesized scale. Chang’s FAHP uses intersection when evaluating the effects of the comparison. This technique is straightforward and easy to adopt. Indeed, the result of the fuzzy intersection can be attained as zero, which indicates that the correlating criterion is irrelevant because the number zero is appointed as weighted. Critics have pointed out that the weights calculated by the extent analysis method do not concern the representation of the relative importance of decision criteria or alternatives. Thus, this could lead to some issues such as low reliability, unjustified priorities, and loss of information. In addition, the extent analysis method might allocate an irrational zero weight to some beneficial decision criteria that will lead them not to be considered in decision-making analysis (Wang et al., 2008).

According to Xu and Liao (2014), there were many researchers such as Chang (1996), Kulak and Kahraman (2005) and Dağdeviren and Yüksel (2008) that did not take into consideration the consistency checking process and this has become a drawback to FAHP method that was developed by them. To overcome this problem, the fuzzy reference relations are converted to the corresponding crisp multiplicative preference relations and then the consistency of the human judgments is checked by using the traditional Saaty’s AHP method (Kwong & Bai, 2003; Chan & Kumar, 2007). Thus, in this paper, the AHP method is used to check the consistency in the decision-making process.

**Materials and Methods**

**Fuzzy Analytic Hierarchy Process (FAHP) Framework**

This study uses the geometric mean FAHP method since the Chang’s (1996) extent analysis method could not make full use of all fuzzy comparison matrix information. Step 1-7 below shows the framework of geometric mean method in FAHP (Buckley, 1985). Prior to the implementation of FAHP method, the AHP method is used to check the consistency of the judgments made by the decision makers by computing the consistency ratio (CR) as $CR = CI/R1$ where $CI$ is the consistency index and $RI$ is the random index (Saaty, 1994). $CI$ is defined as $CI = \frac{\lambda_{max} - n}{n-1}$ where $\lambda_{max}$ is maximum eigenvalue and $n$ is the number of factors in the judgment matrix. The judgments are consistent if the value of $CR < 0.1$.

Step 1: Set up a hierarchy diagram for the problem. The top level of the diagram represents the focus of the detailed problem. The middle level represents the criteria of the focus problem and the bottom level represents the alternative of criteria.

<table>
<thead>
<tr>
<th>AHP Preference Number</th>
<th>AHP Linguistic Variables</th>
<th>TFNs Scale</th>
<th>TFNs Reciprocal Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equally Important (EI)</td>
<td>(1,1,1)</td>
<td>(1,1,1)</td>
</tr>
<tr>
<td>3</td>
<td>Moderately More Important (MMI)</td>
<td>(2,3,4)</td>
<td>(1/4, 1/3, 1/2)</td>
</tr>
<tr>
<td>5</td>
<td>Strongly More Important (SMI)</td>
<td>(4,5,6)</td>
<td>(1/6, 1/5, 1/4)</td>
</tr>
<tr>
<td>7</td>
<td>Very Strong More Important (VSMI)</td>
<td>(6,7,8)</td>
<td>(1/8, 1/7, 1/6)</td>
</tr>
<tr>
<td>9</td>
<td>Extremely More Important (EMI)</td>
<td>(9,9,9)</td>
<td>(1/9, 1/9, 1/9)</td>
</tr>
</tbody>
</table>

Step 2: Scale the data by using the relative scale measurement shown in Table 1 (Ayhan, 2013) and construct the matrix for criteria. Then, the matrix of pairwise comparison is constructed as follows where $\tilde{d}_{ij}^k$ indicates the $k^{th}$ decision maker’s preference of $i^{th}$ criterion over $j^{th}$ criterion, by fuzzy triangular number.
Step 3: If the number of decision maker(s) is more than 1, then the preferences of each
decision maker are averaged as equation below:
\[
\tilde{d}_{ij} = (l_{ij}, m_{ij}, u_{ij}) = \left( \min \left( t_{ij}^k \right), \text{average} \left( m_{ij}^k \right), \max \left( u_{ij}^k \right) \right)
\]  
(2)

The pairwise comparison matrix is therefore been modified as the following matrix:
\[
\tilde{A} = \begin{bmatrix}
\tilde{d}_{11} & \tilde{d}_{12} & \cdots & \tilde{d}_{1n} \\
\tilde{d}_{21} & \ddots & \cdots & \tilde{d}_{2n} \\
\vdots & \ddots & \ddots & \vdots \\
\tilde{d}_{n1} & \cdots & \tilde{d}_{nn}
\end{bmatrix}
\]  
(3)

Step 4: Compute the geometric mean of fuzzy comparison values for each criterion. The
fuzzy geometric mean is provided by equation below:
\[
\tilde{r}_i = \left( \prod_{j=1}^{n} \tilde{d}_{ij} \right)^\frac{1}{n}, i = 1, 2, \ldots, n.
\]  
(4)

Step 5: Calculate the aggregated fuzzy weight of each criterion. The fuzzy weight is
evaluated in equation as follows:
\[
\tilde{w}_i = \tilde{r}_i \oplus (\tilde{r}_1 \oplus \tilde{r}_2 \oplus \cdots \oplus \tilde{r}_n)^{-1} = (l w_i, m w_i, u w_i)
\]  
(5)

Step 6: De-fuzzified the fuzzy weight, \( \tilde{w}_i \) by applying the Centre of Area method, since \( \tilde{w}_i \) are still fuzzy triangular number. The calculation of de-fuzzified can be done by using the
following equation:
\[
M_i = \frac{l w_i + m w_i + u w_i}{3}
\]  
(6)

Step 7: Calculate the relative weight and rank the entire alternative. Since \( M_i \) is non-fuzzy
number it needs to be normalized. The calculation of the relative weight and ranking of the
alternatives are given in equation as follows where \( \sum_{i=1}^{n} N_i = 1 \).
\[
N_j = \frac{M_i}{\sum_{i=1}^{n} M_i}
\]  
(7)

**Implementation of Fuzzy Analytic Hierarchy Process (FAHP) Method**

A real-life empirical data about the selection of the maintenance contractor for a computer
hard disk drive manufacturer and data storage company were used in this study. The company
needs to select the best contractor to handle the machinery maintenance support. The data
about the set of alternatives to be selected and a set of criteria used to select the best
alternative were collected through an interview process with the decision makers. A set of
questionnaires were distributed to five (5) decision makers which are a senior manager, a
manager, an engineer, a senior supervisor, and a specialist. These decision makers are the experts to choose the best contractor for the tasks. There were four (4) alternatives were considered which are contractor A1, A2, A3 and A4 and five (5) chosen criteria which are price (C1), past performance (C2), technical skills (C3), availability (C4) and financial stability (C5).

C2, C3 and C5 were also used by Gholipour, Jandaghi and Rajaei (2014) to select the construction contractor for Tehran University. In this study, all the judgments made by the decision makers are consistent since CR < 0.1. Step 1 – 7 below shows the application of geometric mean in FAHP method:

Step 1: Set up a hierarchy diagram for the problem. The hierarchy diagram of contractor selection problem is shown in Figure 2.

![Hierarchy Diagram for Contractor Selection Problem](image)

Step 2: Scale the matrix by using the relative scale measurement shown in Table 1 and construct the matrix of criteria.

Step 3: Since there were five decision makers, the preferences of each decision maker are averaged by using Eq (2). Calculation below shows how to average the preference for first criterion over second criterion.

\[
\tilde{d}_{ij} = \left( \min \left( \frac{1}{6}, \frac{9}{13}, \frac{4}{8} \right), \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \right), \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \right), \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \right), \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \right), \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \right) = \left( \frac{1}{3}, \frac{1}{9}, \frac{1}{4} \right)
\]

Table 2 below shows the comparison matrix for the criteria after averaged.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>(1,1,1)</td>
<td>(1/9, 3/4, 4)</td>
<td>(1/9, 1/5, 1/2)</td>
<td>(1/6, 16/7, 6)</td>
<td>(1/4, 13/5, 8)</td>
</tr>
<tr>
<td>C2</td>
<td>(1/4, 39/8, 9)</td>
<td>(1,1,1)</td>
<td>(1/4, 7/5, 4)</td>
<td>(2, 23/5,8)</td>
<td>(2, 27/5, 8)</td>
</tr>
<tr>
<td>C3</td>
<td>(2, 29/5, 9)</td>
<td>(1/4, 2, 4)</td>
<td>(1,1,1)</td>
<td>(2, 23/5, 8)</td>
<td>(2, 33/5, 9)</td>
</tr>
<tr>
<td>C4</td>
<td>(1/6, 13/6, 6)</td>
<td>(1/8, 1/4, 1/2)</td>
<td>(1/8, 1/4, 1/2)</td>
<td>(1,1,1)</td>
<td>(1/6, 17/6, 6)</td>
</tr>
<tr>
<td>C5</td>
<td>(1/8, 15/8, 4)</td>
<td>(1/8,2/9,1/2)</td>
<td>(1/9, 1/6, 1/2)</td>
<td>(1/6, 5/4, 6)</td>
<td>(1,1,1)</td>
</tr>
</tbody>
</table>

Step 4: Compute the fuzzy geometric mean. Below is the calculation for the C1 by using Eq (4). Then, the same process is repeated for each criterion.

\[
\tilde{r}_i = \left( \left( \frac{1}{9} \times \frac{1}{6} \times \frac{1}{4} \right)^{\frac{1}{3}}, \left( \frac{3}{4} \times \frac{1}{5} \times \frac{16}{7} \times \frac{13}{5} \right)^{\frac{1}{3}}, \left( \frac{1}{4} \times \frac{1}{2} \times 6 \times 8 \right)^{\frac{1}{3}} \right) = (0.2199, 0.9773, 2.4915)
\]
Table 3 shows the geometric mean of fuzzy comparison values for each criterion \( \bar{r}_i \), vector summation of each \( r_i \) and inverse power of the summation vector in its increasing order.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>( \bar{r}_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>(0.2199, 0.9773, 2.4915)</td>
</tr>
<tr>
<td>C2</td>
<td>(0.7579, 2.7906, 4.7043)</td>
</tr>
<tr>
<td>C3</td>
<td>(1.1487, 3.2093, 4.8164)</td>
</tr>
<tr>
<td>C4</td>
<td>(0.2126, 0.8157, 1.5518)</td>
</tr>
<tr>
<td>C5</td>
<td>(0.1960, 0.6194, 1.4310)</td>
</tr>
<tr>
<td>Total</td>
<td>(2.5351, 8.4123, 14.9950)</td>
</tr>
<tr>
<td>Increasing order</td>
<td>(0.0667, 0.1189, 0.3945)</td>
</tr>
</tbody>
</table>

Step 5: Calculate the fuzzy weight of each criterion, \( \tilde{w}_i \) using Eq (5). Below shows the computation of the fuzzy weight for C1:

\[
\tilde{w}_1 = (0.2199(0.0667), 0.9773(0.1189), 2.4915(0.3945)) = (0.0147, 0.1162, 0.9828)
\]

The same process is repeated for the calculation of fuzzy weight for each criterion. Table 4 shows the fuzzy weight \( \tilde{w}_i \), the weight \( M_i \), the relative weight of \( N_i \) and the rank for each criterion.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Fuzzy Weight ( \tilde{w}_i )</th>
<th>Weight ( M_i )</th>
<th>Relative Weight ( N_i )</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>(0.0147, 0.1162, 0.9828)</td>
<td>0.3712</td>
<td>0.1572</td>
<td>3</td>
</tr>
<tr>
<td>C2</td>
<td>(0.0505, 0.3318, 1.8557)</td>
<td>0.7461</td>
<td>0.3159</td>
<td>2</td>
</tr>
<tr>
<td>C3</td>
<td>(0.0766, 0.3815, 1.8999)</td>
<td>0.7860</td>
<td>0.3329</td>
<td>1</td>
</tr>
<tr>
<td>C4</td>
<td>(0.0142, 0.0970, 0.6122)</td>
<td>0.2411</td>
<td>0.1021</td>
<td>4</td>
</tr>
<tr>
<td>C5</td>
<td>(0.0131, 0.0736, 0.5645)</td>
<td>0.9828</td>
<td>0.0919</td>
<td>5</td>
</tr>
</tbody>
</table>

Step 6: Calculate the weight of each criterion. Below is the calculation of the weight for criteria using Eq (6) and by applying the same equation, the weight acquired for each criterion are \( C_1 = 0.3712, C_2 = 0.7461, C_3 = 0.7860, C_4 = 0.2411 \) and \( C_5 = 0.2171 \) respectively. The weight of each criterion is shown as Table 4.

\[
M_1 = \frac{0.0147 + 0.1162 + 0.9828}{3} = 0.3712
\]

Step 7: Find the relative weight and rank all the criteria. Use Eq (7) to calculate the relative weight of each criterion and alternative. The working example below show the way to calculate the weight on C1:

\[
N_1 = \frac{0.3712}{0.3712 + 0.7461 + 0.7860 + 0.2411 + 0.2171} = 0.1572
\]

The same process is repeated to calculate relative weight for each criterion. The rank of each criterion is shown as Table 4.
Step 2 until step 6 is repeated to calculate the relative weight of alternative and rank the alternative. Next, the final weight of alternatives was calculated by using the arithmetic mean method. Table 5 shows the final weight and rank of the alternative with respect to criterion.

<table>
<thead>
<tr>
<th>Final Weight</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>Weight Rank</th>
<th>Decision makers’ preference</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.3457</td>
<td>0.1070</td>
<td>0.1649</td>
<td>0.1246</td>
<td>0.0809</td>
<td>0.1632</td>
<td>4</td>
</tr>
<tr>
<td>A2</td>
<td>0.3608</td>
<td>0.3166</td>
<td>0.2180</td>
<td>0.2411</td>
<td>0.2067</td>
<td>0.2494</td>
<td>3</td>
</tr>
<tr>
<td>A3</td>
<td>0.1412</td>
<td>0.2573</td>
<td>0.1668</td>
<td>0.3011</td>
<td>0.2976</td>
<td>0.3326</td>
<td>2</td>
</tr>
<tr>
<td>A4</td>
<td>0.1523</td>
<td>0.3192</td>
<td>0.4503</td>
<td>0.3332</td>
<td>0.3148</td>
<td>0.3727</td>
<td>1</td>
</tr>
</tbody>
</table>

Result and Discussion

The findings revealed that ranking order for criterion is given \( C_3 > C_2 > C_1 > C_4 > C_5 \) and Technical skill criterion (\( C_3 \)) is the most preferred criterion used to select the best contractor for the company. The contractor must have high technical skills to be able to cooperate with the company. According to Enshassi et al. (2009), technical skills are needed to understand the integrated plan, schedule and design features of the project, to address the technical problems that may emerge during the project’s design and implementation phases, to understand and coordinate work and to effectively communicate with others from a technical point of view.

On the other hand, the ranking order for alternative is given by \( A_4 > A_3 > A_2 > A_1 \). The findings revealed that Contractor A4 is selected as the most preferred contractor based on the chosen five criteria and the fuzzy preferences of decision makers. As shown in table 6, contractor A4 was selected to handle the maintenance support that year as they were acknowledged by the company to have a skilled team that meets the company’s desirable criteria. Based on their previous work with the company, contractor A4 can finish his tasks in the quoted timeframe and consistently received recognition by the industry. As the result, A4 was selected as the best contractor to provide the services needed by the company.

Conclusion

In this study, FAHP method was used to select the maintenance contractor for a computer hard disk drive manufacturer and data storage company. Contractor plays a critical role in maintenance and services of an equipment in the manufacturing companies (Burhanuddin et al., 2010). Thus, there is a need of formalized decision-making support. The contractor evaluation process is constructed as a MCDM problem under ambiguity, where the imprecise decision-maker's decisions are interpreted as fuzzy numbers (Aydin & Kahraman, 2013). The geometric mean method is used to measure the fuzzy weights for each fuzzy matrices and these are usually merged to obtain the overall fuzzy weights for the alternatives (Aydin & Kahraman, 2013). In addition, this method is more accurate to reflect the uncertainty of decision makers in the decision-making process due to its easiness and computational efficiency. The contractor selection problem can be solved easily by using FAHP method, as the implementation of the method can be carried out in a simple manner. This study will help the company to assess and choose the best contractor based on the chosen criteria that often in conflict. The method can be extended to other selection problem such as supplier selection or portfolio selection. The same problem can also be solved by using other method like Intuitionistic Fuzzy Analytic Hierarchy Process (IFAHP) and Interval-valued Intuitionistic Fuzzy Analytic Hierarchy (IVIFAHP). In this method, the degree of importance of decision makers is taken into considerations and considered as the linguistic variables thus it will make the result to be more reliable.
Acknowledgement
This paper is a work under Universiti Teknologi MARA (UiTM) Negeri Sembilan. We would like to express our gratitude to all UiTM residents, fellow friends, and family.

Conflict of interests
The authors declare that they have no conflicts of interest.

References


